Name: Solutions

Instructor: ______________________

Math 10550, Final Exam:
December 17, 2008

- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.
- No calculators are to be used.
- The exam lasts for two hours.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 14 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)	15.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)	16.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)	17.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)	18.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)	19.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)	20.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)	21.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)	22.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)	23.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)	24.	(a)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(e)	25.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)						
13.	(a)	(b)	(c)	(d)	(e)						
14.	(a)	(b)	(c)	(d)	(e)						

Name: ______________________

Instructor: ______________________

Multiple Choice

1.(6 pts.) Find the limit

$$\lim_{x\to 0}\frac{3-\sqrt{x+9}}{x}.$$

LIMITS

[✓ (a)] $-\frac{1}{6}$

(b) -3

(c) The limit does not exist.

(d) $\frac{1}{6}$

(e) 3

$$\lim_{x\to 0}\frac{3-\sqrt{x+9}}{x} = \lim_{x\to 0}\frac{(3-\sqrt{x+9})}{x}\frac{(3+\sqrt{x+9})}{(3+\sqrt{x+9})}$$

$$= \lim_{x\to 0}\frac{9-(x+9)}{x(3+\sqrt{x+9})} = \lim_{x\to 0}\frac{-x}{x(3+\sqrt{x+9})} = \lim_{x\to 0}\frac{-1}{3+\sqrt{x+9}}$$

$$= \frac{-1}{6}$$

2.(6 pts.) Find all points where the following function is discontinuous

$$f(x)=\begin{cases}\frac{(x-1)(x+2)}{(x^2-1)x} & x\neq 1\\ \frac{3}{2} & x=1\end{cases}.$$

Continuity.

(a) $x=-2, x=-1, x=1$

[✓ (b)] $x=0, x=-1$

(c) $x=0, x=1$

(d) $x=0, x=-2, x=1$

(e) $x=0, x=-1, x=1$

$x = 0$ is not in the domain of f, so f is not continuous at $x=0$.

$(x^2-1) = (x-1)(x+1)$ also appears in the denominator of $\frac{(x-1)(x+2)}{(x^2-1)x}$. $x=-1$ is not in the domain of f, hence f is not continuous at $x=-1$

$f(x)$ is defined at $x=1$; $f(1)=\frac{3}{2}$. For continuity @ $x=1$ we must check if $\lim_{x\to 1} f(x) = f(1)$ i.e. if $\lim_{x\to 1}\frac{(x-1)(x+2)}{(x-1)(x+1)x} = \frac{3}{2}$?

$$\lim_{x\to 1}\frac{\cancel{(x-1)}(x+2)}{\cancel{(x-1)}(x+1)x} = \lim_{x\to 1}\frac{(x+2)}{(x+1)x} = \frac{3}{2} = f(1) \checkmark$$

$\boxed{\text{f is continuous at } x=1}$

Name: ______________________________

Instructor: ___________________________

Chain Rule.

3.(6 pts.) If

$$f(x) = \sqrt{1+\sqrt{1+x}}, \quad = \left(1+\sqrt{1+x}\right)^{\frac{1}{2}}$$

then $f'(8) =$

(a) $\frac{1}{8}$ (b) $\frac{1}{9}$ [✓ (c)] $\frac{1}{24}$ (d) $\frac{1}{2}$ (e) $\frac{1}{12}$

$$f'(x) = \frac{1}{2}\left(1+\sqrt{1+x}\right)^{-1/2} \frac{d}{dx}\left(1+\sqrt{1+x}\right) = \frac{1}{2}\frac{1}{\sqrt{1+\sqrt{1+x}}}\left(\frac{1}{2}(1+x)^{-1/2}\right)$$

$$= \frac{1}{2}\frac{1}{\sqrt{1+\sqrt{1+x}}}\frac{1}{2}\cdot\frac{1}{\sqrt{1+x}}$$

$$f'(8) = \frac{1}{2}\frac{1}{\sqrt{1+\sqrt{9}}}\frac{1}{2}\frac{1}{\sqrt{9}}$$

$$= \frac{1}{4}\frac{1}{\sqrt{4}}\frac{1}{3} = \frac{1}{24}$$

4.(6 pts.) The second derivative of

Quotient/Product Rule

$$f(x) = \frac{\sin x}{x}$$

is

(a) $\dfrac{-x^2\sin x + 4x\cos x + 5\sin x}{x^3}$

(b) $\dfrac{-x^2\sin x - 3x\cos x + 2\sin x}{x^3}$

(c) $\dfrac{x^2\sin x + 4x\cos x + 2\sin x}{x^3}$

[✓ (d)] $\dfrac{-x^2\sin x - 2x\cos x + 2\sin x}{x^3}$

(e) $\dfrac{-x^2\sin x - 3x\cos x + 3\sin x}{x^3}$

$$f'(x) = \frac{x\cos x - \sin x}{x^2}$$

$$f''(x) = \frac{x^2[\cos x + x(-\sin x) - \cos x] - [x\cos x - \sin x]2x}{x^4}$$

$$= \frac{-x^3\sin x - 2x^2\cos x + 2x\sin x}{x^4}$$

$$= \frac{x[-x^2\sin x - 2x\cos x + 2\sin x]}{x^4}$$

$$= \frac{-x^2\sin x - 2x\cos x + 2\sin x}{x^3}$$

Name: ______________________

Instructor: ______________________

Position Function /velocity.

Application of Differentiation

5.(6 pts.) A body travels along a straight line according to the law

$$s = -t^4 - 4t^3 + 20t^2, \quad t \geq 0.$$

At what position, **after** the motion gets started, does the body first come to rest?

(a) $s = 36$ (b) $s = 24$ (c) $s = 2$

[✓ (d)] $s = 32$ (e) $s = 12$

Body is at rest when $\frac{ds}{dt} = 0$ i.e. when $-4t^3 - 12t^2 + 40t = 0$

or $-4t(t^2 + 3t - 10) = 0$

or $-4t(t-2)(t+5) = 0$

i.e. $t = 0$ $t = 2$ or $t = -5$

$t \geq 0$ and we have ruled out $t = 0$

So $t = 2$ gives the time when the body comes to rest for the first time after $t = 0$

when $t = 2$, $s = -(2)^4 - 4(2)^3 + 20(2)^2$

$= -16 - 32 + 80$

$= 32$.

6.(6 pts.) Find an equation for the tangent line to

$$f(x) = \tan(x^2 + 2x)$$

at the point $(0, 0)$.

Tangents + chain Rule.

[✓ (a)] $y = 2x$ (b) $y = 0$ (c) $y = \sqrt{2}x$

(d) $y = 2\sqrt{2}x$ (e) $y = -2x$

$(0,0)$ is on line. and slope $= f'(0)$.

$f'(x) = \left[\sec^2(x^2 + 2x)\right](2x + 2)$

using chain rule.

$f'(0) = \left[\sec^2(0)\right] 2 = 2\frac{1}{\cos^2 0} = 2.$

Equation of tangent at $(0,0)$ is $y - 0 = m(x - 0)$

$y = 2x.$

Name: ________________________

Instructor: ________________________

7.(6 pts.) Find an equation for the tangent line to the curve Implicit DifferENTIATION

$$x^3 + y^3 = 4xy$$

at the point (2, 2).

(a) $y = 2x - 2$ (b) $y = x$ [✓ (c)] $y = -x + 4$

(d) $y = -x - 4$ (e) $y = -2x + 6$

We must find $y' = \frac{dy}{dx}$ when $x=2$ and $y=2$.

we differentiate both sides of the above equation.

$$3x^2 + 3y^2y' = 4[y + xy']$$

when $x=2, y=2 \to 3(2)^2 + 3(2)^2y' = 4[2 + 2y']$

ie. $12 + 12y' = 8 + 8y'$

or $4y' = -4 \to y' = -1$

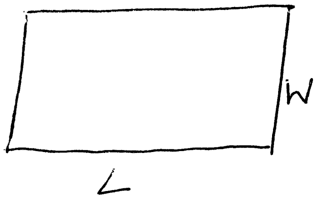
AT $(2,2)$, Equation of Tangent: $(y-2) = m(x-2) \to y - 2 = -1(x-2)$

$$\boxed{y = -x + 4}$$

8.(6 pts.) The length of a rectangle is increasing at a rate of 8 cm/sec and its width is increasing at a rate of 3 cm/sec. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing? RELATED RATES

[✓ (a)] 140 cm^2/sec. (b) 211 cm^2/sec. (c) 190 cm^2/sec.

(d) 11 cm^2/sec. (e) 24 cm^2/sec.



$A = LW$

$\frac{dA}{dt} = W\frac{dL}{dt} + L\frac{dW}{dt}$.

when $L = 20$ and $W = 10$

$\frac{dA}{dt} = 10\frac{dL}{dt} + 20\frac{dW}{dt} = 10 \cdot 8 + 20 \cdot 3$

$= 80 + 60 = 140\ \text{cm}^2/\text{s}$

Given: $\frac{dL}{dt} = 8$ cm/s

$\frac{dW}{dt} = 3$ cm/s

To Find $\frac{dA}{dt}$ when

$L = 20$ and $W = 10$

Name: ______________________

Instructor: ______________________

LINEAR APPROXIMATION

9.(6 pts.) Use linear approximation to estimate

$$\frac{1}{\sqrt{3.9}}.$$

(a) $\frac{1}{\sqrt{3.9}} \approx \frac{9}{20}$ (b) $\frac{1}{\sqrt{3.9}} \approx \frac{1}{2}$ [✓ (c)] $\frac{1}{\sqrt{3.9}} \approx \frac{81}{160}$

(d) $\frac{1}{\sqrt{3.9}} \approx \frac{11}{20}$ (e) $\frac{1}{\sqrt{3.9}} \approx \frac{79}{160}$

Let $f(x) = \frac{1}{\sqrt{x}}$ $f(4) = \frac{1}{\sqrt{4}} = \frac{1}{2}$ Let $a = 4$

$$\underbrace{L(x) \approx f(a) + f'(a)(x-a)}_{\text{Linearization at } a} = \underbrace{f(4) + f'(4)(x-4)}_{\text{Linearization at } 4} = \frac{1}{2} - \frac{1}{16}(x-4)$$

$f'(x) = -\frac{1}{2}x^{-3/2} = -\frac{1}{2}\frac{1}{x^{3/2}}$ $f'(4) = -\frac{1}{2}\frac{1}{4\sqrt{4}} = -\frac{1}{16}$

For x near 4 $f(x) \approx L(x) = \frac{1}{2} - \frac{1}{16}(x-4)$

For $x = 3.9$ $f(3.9) = \frac{1}{\sqrt{3.9}} \approx L(3.9) = \frac{1}{2} - \frac{1}{16}(3.9-4) = \frac{1}{2} - \frac{1}{16}(-.1)$
$= \frac{1}{2} + \frac{1}{160} = \frac{81}{160}$

10.(6 pts.) Let

$$f(x) = x^3 + 3x^2 - 24x.$$

Find the absolute maximum and absolute minimum values of f on the interval $[0, 10]$.

Absolute max/min
EXTREME VALUE
THEOREM.

(a) Max at $x = 4$; Min at $x = 0$. (b) Max at $x = 10$; Min at $x = 0$.

(c) Max at $x = 4$; Min at $x = 2$. [✓ (d)] Max at $x = 10$; Min at $x = 2$.

(e) Max at $x = 4$; Min at $x = 1$.

Absolute max/min must occur at end points or at a critical point in the interval $[0,10]$, since $f(x)$ is continuous on the interval $[0,10]$

Critical Points: where $f'(x) = 0$ i.e. $3x^2 + 6x - 24 = 0$

or $3(x^2 + 2x - 8) = 0$

or $3(x-2)(x+4) = 0$

$\rightarrow x = 2$ or $x = -4$

$x = 2$ is in the interval $[0,10]$.

We check for max/min

	f(x)
0	0
2	8+12-48=-28 MIN
10	1000+3(100)-240 = 1060 Max

Name: ______________________________

Instructor: ___________________________

11.(6 pts.) Find the local and absolute maximum and minimum of

$$f(x) = 3x^{2/3} - x.$$

Domain f = all $\mathbb{R}$.

(a) Local min at $x = 1/8$; absolute min at $x = 1$; no absolute max.

(b) Local min at $x = 1$; local max at $x = 1/8$; no absolute min; absolute max at $x = -27$.

(c) Absolute min at $x = 0$; absolute max at $x = 8$.

[✓ (d)] Local min at $x = 0$; local max at $x = 8$; no absolute max or min.

(e) Local max at $x = 1$; no absolute max; absolute min at $x = 0$.

$f'(x) = \frac{2}{3} \cdot 3x^{-1/3} - 1 = \frac{2}{x^{1/3}} - 1.$

CRITICAL POINTS

$f'(x) = 0$ if $\frac{2}{x^{1/3}} - 1 = 0$ or $\frac{2}{x^{1/3}} = 1 \to x^{1/3} = 2 \downarrow \boxed{x = 8}$

$f'(x)$ D.N.E if $\boxed{x = 0}$

$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \underset{-\infty}{\underset{\downarrow}{x}}(\underset{-1}{\underset{\downarrow}{3x^{-1/3} - 1}}) = +\infty$

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \underset{\infty}{\underset{\downarrow}{x}}(\underset{-1}{\underset{\downarrow}{3x^{-1/3} - 1}}) = -\infty$

NO ABS. MAX OR MIN

Dec		inc.		Dec	$f(x)$
−		+		−	Sign f'
$f'(-1) = -3 < 0$	0	$f'(1) = 2-1 = 1 > 0$	8	$f'(27) = \frac{2}{3} - 1 < 0$	
	LOCAL MIN		LOCAL MAX		

12.(6 pts.) Let

$$f(x) = x^{5/3} - 5x^{2/3}.$$

On what intervals is f concave up?

[✓ (a)] $(-1, 0) \cup (0, \infty)$ (b) $(-8, 8)$ (c) $(1, \infty)$

(d) $(-\infty, -1)$ (e) $(0, 8)$

$f'(x) = \frac{5}{3}x^{2/3} - \frac{10}{3}x^{-1/3}$ $f''(x) = \frac{10}{9}x^{-1/3} - \left(\frac{10}{9}(-1)\right)x^{-4/3} = \frac{10}{9}\left(\frac{1}{x^{1/3}} + \frac{1}{x^{4/3}}\right)$

$= \frac{10}{9}\left(\frac{x+1}{x^{4/3}}\right)$

$f''(x)$ DNE if $x = 0$

$f''(x) = 0$ if $x = -1$

−		+		+	$x+1$
+		+		+	$x^{4/3}$
−	−1	+	0	+	$f''(x)$
Conc. down.		Conc up.		conc up.	$f(x)$

Name: ______________________________

Instructor: ___________________________

13.(6 pts.) Evaluate the limit

$$\lim_{x\to\infty}\left(\sqrt{x^2+2x}-x\right).$$

(a) $-\infty$ (b) 0 [✓ (c) 1] (d) 2 (e) ∞

$$\lim_{x\to\infty}\left(\sqrt{x^2+2x}-x\right)=\lim_{x\to\infty}\left(\sqrt{x^2+2x}-x\right)\frac{\left(\sqrt{x^2+2x}+x\right)}{\left(\sqrt{x^2+2x}+x\right)}=$$

$$\lim_{x\to\infty}\frac{(\cancel{x^2}+2x)-\cancel{x^2}}{\sqrt{x^2+2x}+x}=\lim_{x\to\infty}\frac{2x}{\sqrt{x^2+2x}+\cancel{\infty}}=\lim_{x\to\infty}\frac{2x/x}{\left(\sqrt{x^2+2x}+x\right)/x}$$

$$=\lim_{x\to\infty}\frac{2}{\sqrt{\frac{x^2+2x}{x^2}}+1}=\lim_{x\to\infty}\frac{\overset{\to 2}{2}}{\underset{\to 2}{\sqrt{1+\frac{2}{x}}+1}}=1$$

14.(6 pts.) The equation of the slant asymptote of the curve $y=\dfrac{2x^2+1}{x+1}$ is:

(a) $y=2x$ [✓ (b) $y=2x-2$] (c) $y=-2x+2$

(d) $y=x+2$ (e) $y=2x+2$

$$\begin{array}{r|l} & 2x-2 \\ \hline x+1 & 2x^2+1 \\ & \ominus\cancel{2x^2}\ \ominus\!+2x \\ \hline & 1-\cancel{2x} \\ & \oplus\!-2\ \ \oplus\!-\cancel{2x} \\ \hline & 1 \end{array}$$

$$2x^2+1=(2x-2)(x+1)+1$$

$$\frac{2x^2+1}{x+1}=2x-2+\frac{1}{x+1}$$

$$\lim_{x\to\pm\infty}\left(\frac{2x^2+1}{x+1}-(2x-2)\right)=\lim_{x\to\pm\infty}\frac{1}{x+1}$$

$$=0.$$

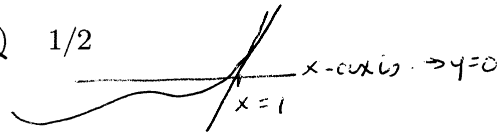
SLANT Asymptote $y=2x-2.$

Name: ______________________

Instructor: ______________________

15.(6 pts.) Suppose the line $y = 4x - 2$ is tangent to the curve $y = f(x)$, when $x = 1$. If the Newton's method is used to locate a root of the equation $f(x) = 0$ and the initial approximation is $x_1 = 1$, find the second approximation x_2

(a) -4 (b) 1 (c) 0 (d) 2 [✓ (e)] 1/2



Easy Method

X_2 = point where tangent cuts x-axis

ie $0 = 4X - 2$ or $4X = 2$ or $\boxed{X_2 = \frac{1}{2}}$

Difficult Method.

If you wish to use the Formula. $X_2 = X_1 - \frac{f(x_1)}{f'(x_1)} = 1 - \frac{f(1)}{f'(1)}$

y-value of tangent = y-value of f(x) at $x_1 = 1$

$= 4 - 2 = 2$. $f(1) = 2$

$4 =$ slope of tangent at $x_1 = 1$ equals $f'(1)$.

$f'(1) = 4$

↓

$\boxed{X_2 = 1 - \frac{2}{4} = \frac{1}{2}}$

16.(6 pts.) Calculate the following definite integral

$$\int_1^5 (5 - x)^2 dx =$$

(a) 16 (b) $-\frac{64}{3}$ (c) 3 (d) -16 [✓ (e)] $\frac{64}{3}$

Can either expand $(5-x)^2$ or use substitution

let $u = 5 - x$ $du = -1 \cdot dx$ ⇒ $dx = -du$. $u(1) = 4$ $u(5) = 0$

$$\int_1^5 (5-x)^2 dx = \int_4^0 u^2(-1)du = -\int_4^0 u^2 du = +\int_0^4 u^2 du$$

↓

From laws of integration

$$= \frac{u^3}{3}\Big|_0^4 = \frac{64}{3} - 0 = \boxed{\frac{64}{3}}$$

Name: ______________________________

Instructor: ___________________________

17.(6 pts.) Let $g(x) = \int_{\sin x}^{0} t^2\, dt$. Find $g'(x)$.

(a) $-(\cos x)^2 \cos x$

[✓ (b)] $-(\sin x)^2 \cos x$

(c) $(\cos x)^2 \cos x$

(d) $-(\sin x)^2 \sin x$

(e) $(\sin x)^2 \cos x$

$$\frac{d}{dx}\int_{\sin x}^{0} t^2\,dt = -\frac{d}{dx}\int_{0}^{\sin x} t^2\,dt = -\left[\frac{d}{du}\int_{0}^{u} t^2\,dt\right]\frac{du}{dx}$$

$$= -u^2\frac{du}{dx} = -(\sin^2 x)\cos x$$

18.(6 pts.) Calculate the integral $\int_0^2 \frac{x}{\sqrt{x^2+1}}\,dx$

[✓ (a)] $\sqrt{5}-1$

(b) $-\sqrt{5}-1$

(c) $1-\sqrt{5}$

(d) $\sqrt{5}$

(e) 4

let $u = x^2+1$ $\quad du = 2x\,dx \to x\,dx = \frac{du}{2}$

$u(0) = 1 \quad u(2) = 5.$

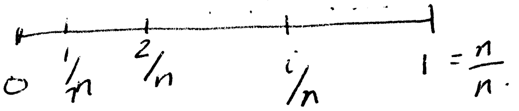
$$\int_0^2 \frac{x\,dx}{\sqrt{x^2+1}} = \frac{1}{2}\int_1^5 \frac{du}{\sqrt{u}} = \frac{\cancel{1}}{\cancel{2}}\left[\frac{u^{1/2}}{\cancel{1/2}}\right]_1^5 = \sqrt{5}-\sqrt{1} = \sqrt{5}-1$$

Name: ____________________

Instructor: ____________________

19.(6 pts.) Which of the following is a Riemann sum corresponding to the integral

$$\int_0^1 (\tan x + 2)dx.$$



(a) $2 + \frac{1}{n}\sum_{i=1}^{n} \tan(\frac{i}{n})$

(b) $\frac{2}{n} + \frac{2}{n}\sum_{i=1}^{n} \tan(\frac{i}{n})$

[✓ (c)] $\frac{1}{n}\sum_{i=1}^{n}\left(\tan(\frac{i}{n}) + 2\right)$

(d) $\frac{2}{n}\sum_{i=1}^{n} \tan(\frac{2i}{n})$

(e) $\frac{1}{2n}\sum_{i=1}^{n} \tan(\frac{2i}{n})$

$f(x) = \tan x + 2$

$f(\frac{i}{n}) = \tan(\frac{i}{n}) + 2$

$\Delta x = \frac{1}{n}$

$$\sum_{i=1}^{n} f(\tfrac{i}{n}) \Delta x$$

$$= \sum_{i=1}^{n} \left(\tan(\tfrac{i}{n}) + 2\right)\frac{1}{n}$$

$$= \frac{1}{n}\sum_{1}^{n}\left(\tan(\tfrac{i}{n}) + 2\right)$$

20.(6 pts.) The point on the line $6x + y = 9$ that is closest to the origin has x-coordinate

(a) $x = \frac{3}{2}$

(b) $x = 0$

(c) $x = 1$

(d) $x = \frac{44}{9}$

[✓ (e)] $x = \frac{54}{37}$

Each point on the line $y = 9 - 6x$ has coordinates $(x, 9-6x)$. The distance to the origin for such a point is $D = \sqrt{x^2 + (9-6x)^2}$. D is at a minimum when its square is at a minimum ie when $f(x) = x^2 + (9-6x)^2$ is at a minimum. $f'(x) = 2x + 2(9-6x)(-6) = 2x - 108 + 72x$
$= 74x - 108$

$f'(x) = 0$ if $74x = 108$ or $\boxed{x = \frac{54}{37}}$

$f'(x) < 0$ for $x < \frac{54}{37}$ and $f'(x) > 0$ if $x > \frac{54}{37}$ Therefore f has a min at this point

Name: ______________________

Instructor: ______________________

21.(6 pts.) The curves $y = x^4 - 3$ and $y = -x^4 + 5$ enclose an area. Set up a definite integral which calculates the area of this region.

(a) $\int_{-1}^{1} (8 - 2x^4)\,dx$

(b) $\int_{0}^{\sqrt{3}} (8 - 2x^4)\,dx$

(c) $\int_{-1}^{1} 2\,dx$

(d) $\int_{-\sqrt{2}}^{\sqrt{2}} 2\,dx$

[✓ (e)] $\int_{-\sqrt{2}}^{\sqrt{2}} (8 - 2x^4)\,dx$

Curves meet when

$x^4 - 3 = -x^4 + 5$

or $2x^4 = 8$

or $x^4 = 4$.

or $x = \pm\sqrt{2}$.

To check which one is Larger on the interval $[-\sqrt{2}, \sqrt{2}]$. it is enough to check at a point since both functions are continuous.

at $x = 0 \Rightarrow x^4 - 3 - (-x^4 + 5) = -3 - 5 = -8 < 0$

Hence $-x^4 + 5 > x^4 - 3$ on the interval $(-\sqrt{2}, \sqrt{2})$

and area btwn. curves is given by $\int_{-\sqrt{2}}^{\sqrt{2}} -x^4 + 5 - (x^4 - 3)\,dx = (e)$

22.(6 pts.) The plane region bounded below by the graph of $y = x$ and above by the graph $y = \sqrt{x}$ is rotated about the line $x = 5$. Which integral below gives the volume?

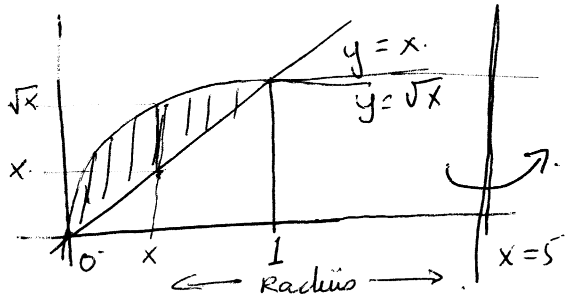
(a) $\pi \int_{0}^{1} (5 - \sqrt{x})^2 - (5 - x)^2 dx$

(b) $\pi \int_{0}^{1} (5 - x)^2 - (5 - \sqrt{x})^2 dx$

(c) $2\pi \int_{0}^{1} (x - 5) \cdot (\sqrt{x} - x) dx$

(d) $2\pi \int_{0}^{1} (5 - x) \cdot (x - \sqrt{x}) dx$

[✓ (e)] $2\pi \int_{0}^{1} (5 - x) \cdot (\sqrt{x} - x) dx$



$\sqrt{x} = x$ if $x = x^2$ or $x(x-1) = 0$

i.e $x = 0, 1$

shell method.

$$V = 2\pi \int_{0}^{1} \underset{\text{Radius}}{(5-x)} \underset{\text{height}}{(\sqrt{x} - x)}\,dx.$$

to see which is Larger it is enough to check at a single point in the interval since both functions are continuous

We check value of $\sqrt{x} - x$ @ $x = \frac{1}{4}$

$x = \frac{1}{4} \Rightarrow \sqrt{x} - x = \frac{1}{2} - \frac{1}{4} = \frac{1}{4} > 0$

so $\sqrt{x} > x$ on interval $(0, 1)$

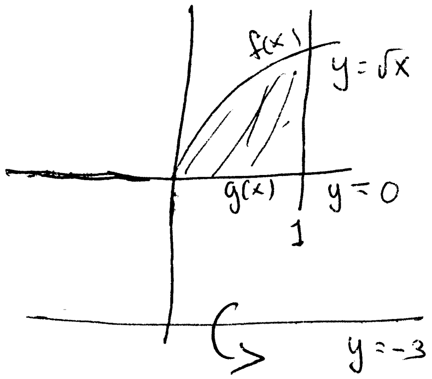
Actually This is given in statement of problem. →

Name: ______________________________

Instructor: ___________________________

23.(6 pts.) Consider the plane region bounded by the graphs of $y = \sqrt{x}$, $y = 0$ and $x = 1$. Rotate this region about the line $y = -3$ and calculate the volume.

(a) $\frac{3\pi}{3}$ [✓ (b) $\frac{9\pi}{2}$] (c) $\frac{7\pi}{2}$ (d) $\frac{15\pi}{2}$ (e) $\frac{27\pi}{2}$



Using method of Washers.

$$V = \int_0^1 \pi \left[\underbrace{(f(x) - (-3))^2}_{\text{OUTER Radius}} - \underbrace{(g(x) - (-3))^2}_{\text{INNER Radius}}\right] dx$$

$$= \pi \int_0^1 \left[(\sqrt{x} + 3)^2 - (0 + 3)^2\right] dx$$

$$= \pi \int_0^1 x + 6\sqrt{x} + 9 - 9 \, dx$$

$$= \pi \left[\frac{x^2}{2} + \frac{6x^{3/2}}{3/2}\right]_0^1$$

$$= \pi\left[\frac{1}{2} + 4\right] = \frac{\pi 9}{2}.$$

24.(6 pts.) Find the average of $f(x) = \sin^2(x) \cdot \cos(x)$ over $[0, \frac{\pi}{2}]$.

[✓ (a) $\frac{2}{3\pi}$] (b) $\frac{2}{\pi}$ (c) $\frac{1}{3}$

(d) $\frac{1}{\pi}$ (e) $\frac{1}{3\pi}$

$$f_{ave} = \frac{1}{\pi/2} \int_0^{\pi/2} \sin^2(x) \cdot \cos(x) \, dx$$

let $u = \sin x$ $du = \cos x \, dx$ $u(0) = 0$

$u(\frac{\pi}{2}) = 1$

$$f_{ave} = \frac{2}{\pi} \int_0^1 u^2 du = \frac{2}{\pi}\left[\frac{u^3}{3}\right]_0^1 = \frac{2}{\pi}\frac{1}{3} = \frac{2}{3\pi}$$

Name: ______________________________

Instructor: ___________________________

25.(6 pts.) A (vertical) cylindrical tank has a height 1 meter and base radius 1 meter. It is filled full with a liquid with a density 100 kg/m^3. Find the work required to empty the tank by pumping all of the liquid to the top of the tank.

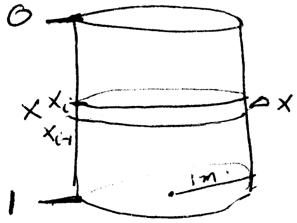
(a) 0 kg-m

(b) 200π kg-m

(c) ~~50π kg-m~~ 490π J

(d) 500π kg-m

(e) 100π kg-m



W_i = work done on slice $[x_{i-1}, x_i]$

$= F_i \cdot d_i$ (Force x distance).

F_i = Volume of slice $\times 100 \times g$.

where $g = 9.8\ m/s^2$.

$= \pi (r^2) \Delta x (980)$

$= \pi\ 980\ \Delta x$ N

d_i = distance $\simeq x_i$.

Work ~~a~~ required $\simeq W_1 + W_2 + \cdots + W_n$.

$$= \sum_{i=1}^{n} 980\pi(\Delta x) x_i$$

$$= 980\pi \sum_{i=1}^{n} x_i \Delta x.$$

Work required $= \lim_{n \to \infty} \sum_{i=1}^{n} 980\pi\ x_i \Delta x$

$$= \int_0^1 980\pi\ x\, dx$$

$$= 980\pi \left[\frac{x^2}{2}\right]_0^1 = \frac{980\pi}{2}$$

$= 490\pi$ J

Name: ______________________

Instructor: ANSWERS

Math 10550, Final Exam:
December 17, 2008

- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.
- No calculators are to be used.
- The exam lasts for two hours.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 14 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1. (•) (b) (c) (d) (e)
2. (a) (•) (c) (d) (e)

3. (a) (b) (•) (d) (e)
4. (a) (b) (c) (•) (e)

5. (a) (b) (c) (•) (e)
6. (•) (b) (c) (d) (e)

7. (a) (b) (•) (d) (e)
8. (•) (b) (c) (d) (e)

9. (a) (b) (•) (d) (e)
10. (a) (b) (c) (•) (e)

11. (a) (b) (c) (•) (e)
12. (•) (b) (c) (d) (e)

13. (a) (b) (•) (d) (e)
14. (a) (•) (c) (d) (e)

15. (a) (b) (c) (d) (•)
16. (a) (b) (c) (d) (•)

17. (a) (•) (c) (d) (e)
18. (•) (b) (c) (d) (e)

19. (a) (b) (•) (d) (e)
20. (a) (b) (c) (d) (•)

21. (a) (b) (c) (d) (•)
22. (a) (b) (c) (d) (•)

23. (a) (•) (c) (d) (e)
24. (•) (b) (c) (d) (e)

25. (a) (b) (•) (d) (e)